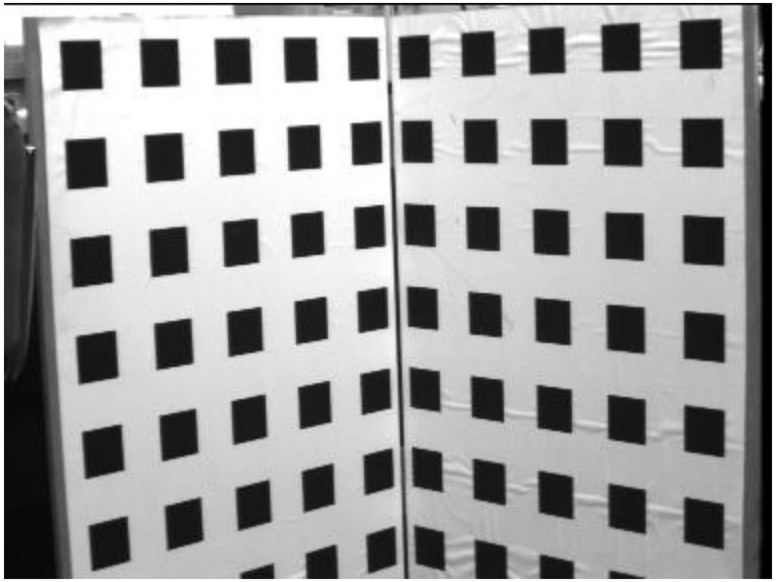
Camera Calibration (Compute Camera Matrix P)

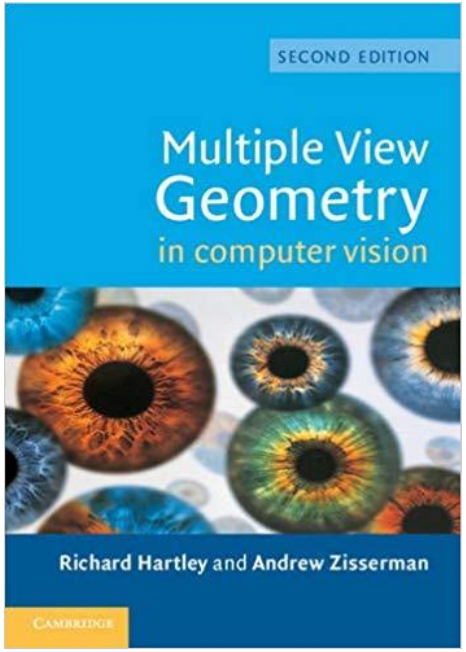
簡韶逸 Shao-Yi Chien
Department of Electrical Engineering
National Taiwan University

Fall 2018

Outline

- Camera calibration

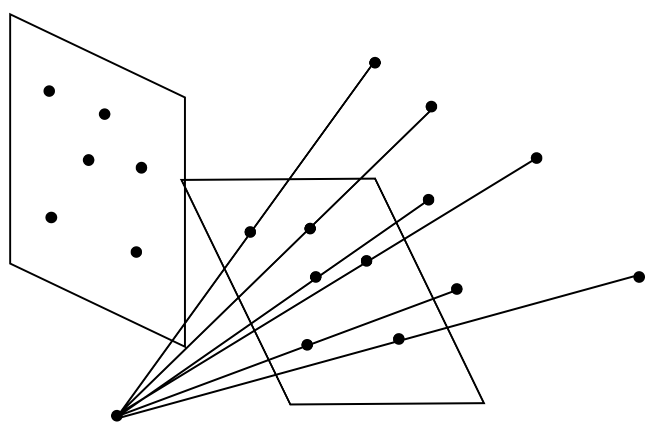




[Slides credit: Marc Pollefeys]

Resectioning

$$X_i \leftrightarrow x_i \qquad P\,?$$



Basic Equations

$$\mathrm{x}_i = \mathrm{P}\mathrm{X}_i$$

$$[\mathrm{x}_i]_\times \mathrm{P}\mathrm{X}_i$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i \mathbf{X}_i{}^\top & y_i \mathbf{X}_i{}^\top \\ w_i \mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i \mathbf{X}_i{}^\top \\ -y_i \mathbf{X}_i{}^\top & x_i \mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i \mathbf{X}_i{}^\top & y_i \mathbf{X}_i{}^\top \\ w_i \mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i \mathbf{X}_i{}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$\mathrm{Ap} = 0$$

Basic Equations

$$\mathrm{Ap} = 0$$

minimal solution

P has 11 dof, 2 independent eq./points

$\Rightarrow$ 5½ correspondences needed (say 6)

Over-determined solution

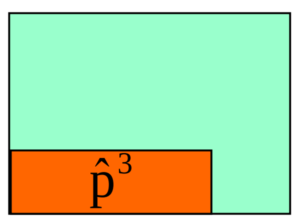
$n \geq$ 6 points

minimize $\left\| \mathrm{Ap} \right\|$ subject to constraint

$$\left\| \mathrm{p} \right\| = 1$$

or $\left\| \hat{\mathrm{p}}^3 \right\| = 1$

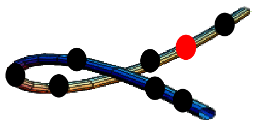
$\mathrm{P} =$



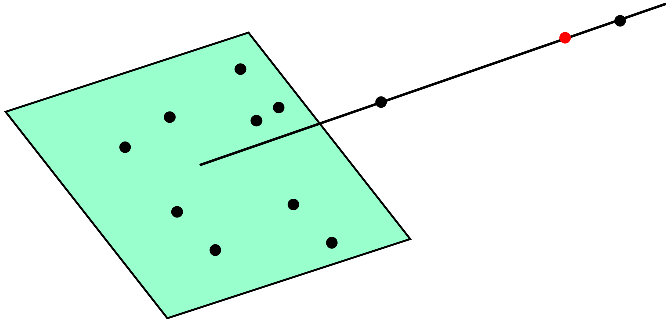
Degenerate Configurations

More complicate than 2D case

(i) Camera and points on a twisted cubic



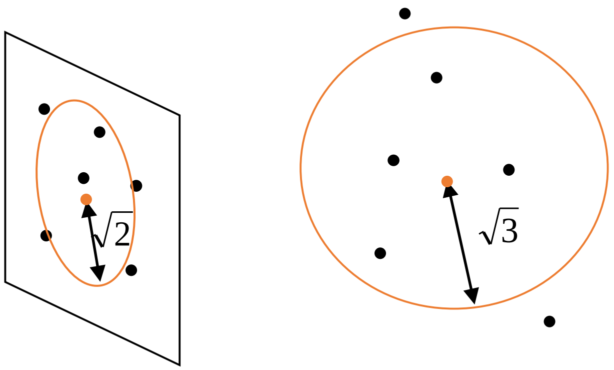
(ii) Points lie on plane or single line passing through projection center



Data Normalization

Less obvious

(i) Simple, as before



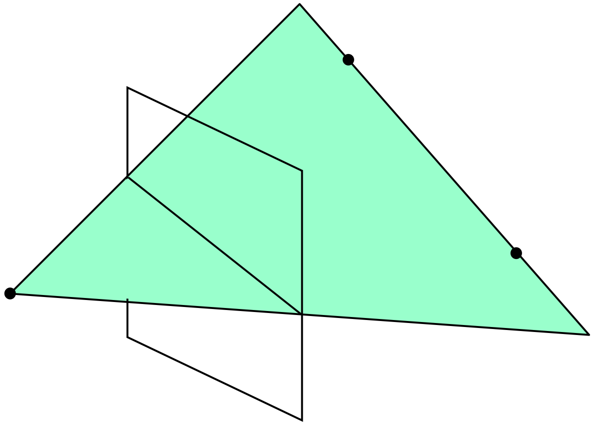
(ii) Anisotropic scaling

Line Correspondences

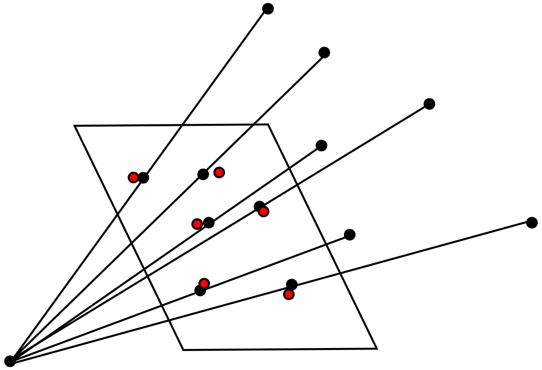
Extend DLT to lines

$$\Pi = P^T l_i \qquad \text{(back-project line)}$$

$$l_i^T P X_{1i} \qquad l_i^T P X_{2i} \qquad \text{(2 independent eq.)}$$



Geometric Error



$$\sum_i d(\mathbf{x}_i, \hat{\mathbf{x}}_i)^2$$

$$\min_{\mathrm{P}} \sum_i d(\mathbf{x}_i, \mathrm{P}\mathbf{X}_i)^2$$

Gold Standard Algorithm

Objective

Given n≥6 2D to 2D point correspondences $\{X_i \leftrightarrow x_i'\}$, determine the Maximum Likelyhood Estimation of P

Algorithm

(i) Linear solution:

(a) Normalization: $\tilde{X}_i = UX_i \quad \tilde{x}_i = Tx_i$

(b) DLT:

(ii) Minimization of geometric error: using the linear estimate as a starting point minimize the geometric error:

$$\min_{\tilde{P}} \sum_i d(\tilde{\mathbf{x}}_i, \tilde{P}\tilde{\mathbf{X}}_i)^2$$

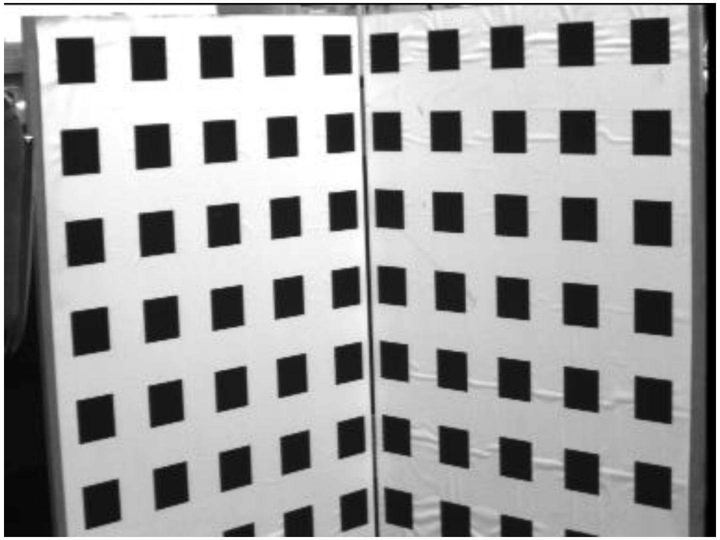
(iii) Denormalization: $P = T^{-1}\tilde{P}U$

Calibration Example

(i) Canny edge detection
(ii) Straight line fitting to the detected edges
(iii) Intersecting the lines to obtain the images corners

typically precision <1/10

(HZ rule of thumb: 5*n* constraints for *n* unknowns



	f_y	f_x/f_y	skew	x_0	y_0	residual
linear	1673.3	1.0063	1.39	379.96	305.78	0.365
iterative	1675.5	1.0063	1.43	379.79	305.25	0.364

Errors in the World

$$\sum_i d(\mathbf{X}_i, \widehat{\mathbf{X}}_i)^2$$

$$\mathrm{x}_i = \mathrm{P}\widehat{\mathrm{X}}_i$$

Errors in the image and in the world

$$\sum_{i=1}^{n} d_{\mathrm{Mah}}(\mathbf{x}_i, \mathrm{P}\widehat{\mathbf{X}}_i)^2 + d_{\mathrm{Mah}}(\mathbf{X}_i, \widehat{\mathbf{X}}_i)^2$$

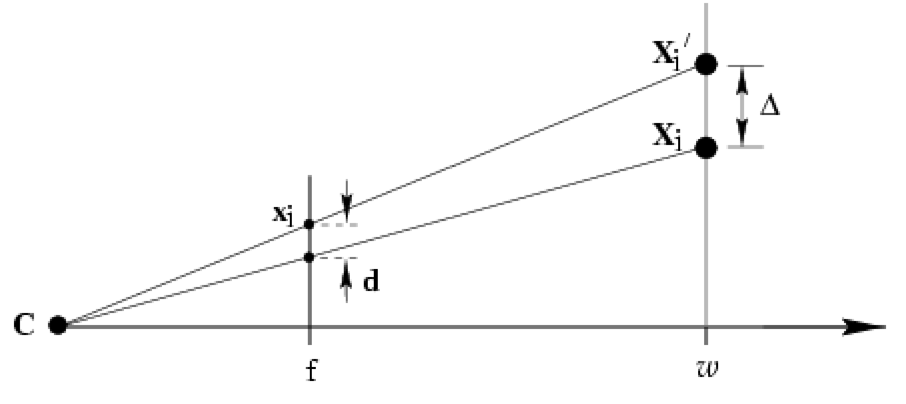
Geometric Interpretation of Algebraic error

$$\sum_i \left(\hat{w}_i d(\mathrm{x}_i, \hat{\mathrm{x}}_i)\right)^2$$

$$\hat{w}_i\left(\hat{x}_i, \hat{y}_i, 1\right) = \mathrm{P}\mathrm{X}_i \qquad \hat{w}_i = \pm\left\|\hat{\mathrm{p}}^3\right\|\mathrm{depth}(\mathrm{X};\mathrm{P})$$

$$\text{therefore, if } \left\|\hat{\mathrm{p}}^3\right\| = 1 \text{ then}$$

$$\hat{w}_i d(\mathrm{x}_i, \hat{\mathrm{x}}_i) \sim f d(\mathrm{X}_i, \hat{\mathrm{X}}_i)$$



Estimation of Affine Camera

Last row = (0, 0, 0, 1)

$$\begin{bmatrix} \mathbf{0}^\top & -w_i \mathbf{X}_i{}^\top & y_i \mathbf{X}_i{}^\top \\ w_i \mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i \mathbf{X}_i{}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{0}^\top & -\mathbf{X}_i{}^\top \\ \mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \end{pmatrix} + \begin{pmatrix} y_i \\ -x_i \end{pmatrix} = \mathbf{0}$$

$$||\mathtt{A}\mathbf{p}||^2 = \sum_i \left(x_i - \mathbf{P}^{1\top}\mathbf{X}_i\right)^2 + \left(y_i - \mathbf{P}^{2\top}\mathbf{X}_i\right)^2 = \sum_i d(\mathbf{x}_i, \hat{\mathbf{x}}_i)^2$$

note that in this case algebraic error = geometric error

Gold Standard Algorithm

Objective

Given n≥4 2D to 2D point correspondences $\{X_i \leftrightarrow x_i'\}$, determine the Maximum Likelyhood Estimation of P

(remember $P^{3T}=(0,0,0,1)$)

Algorithm

(i) Normalization: $\tilde{X}_i = UX_i \quad \tilde{x}_i = Tx_i$

(ii) For each correspondence

$$\begin{bmatrix} \mathbf{0}^\top & -\mathbf{X}_i{}^\top \\ \mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \end{pmatrix} + \begin{pmatrix} y_i \\ -x_i \end{pmatrix} = \mathbf{0}$$

$$A_8 p_8 = b$$

(iii) solution is

$$p_8 = A_8{}^+ b$$

(iv) Denormalization: $P = T^{-1}\tilde{P}U$

Restricted Camera Estimation

Find best fit that satisfies

- skew s is zero
- pixels are square
- principal point is known
- complete camera matrix K is known

$$
\mathrm{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}
$$

Minimize geometric error

$\rightarrow$impose constraint through parametrization

$\rightarrow$Image only $\mathbb{R}^9 \rightarrow \mathbb{R}^{2n}$, otherwise $\mathbb{R}^{3n+9} \rightarrow \mathbb{R}^{5n}$

Minimize algebraic error

$\rightarrow$assume map from param q $\rightarrow$ P=K[R|-RC], i.e. p=g(q)

$\rightarrow$minimize ||Ag(q)||

Reduced Measurement Matrix

One only has to work with 12x12 matrix, not 2nx12

$$\|Ap\| = p^T A^T Ap = \|\hat{A}p\|$$

$$A^\top A = (VDU^\top)(UDV^\top) = (VD)(DV^\top) = \hat{A}^\top \hat{A}$$

Restricted Camera Estimation

$$\mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

Initialization

- Use general DLT
- Clamp values to desired values, e.g. s=0, $\alpha_x = \alpha_y$

Note: can sometimes cause big jump in error

Alternative initialization

- Use general DLT
- Impose soft constraints

$$\sum_i d(\mathbf{x}_i, \mathrm{P}\mathbf{X}_i)^2 + ws^2 + w(\alpha_x - \alpha_y)^2$$

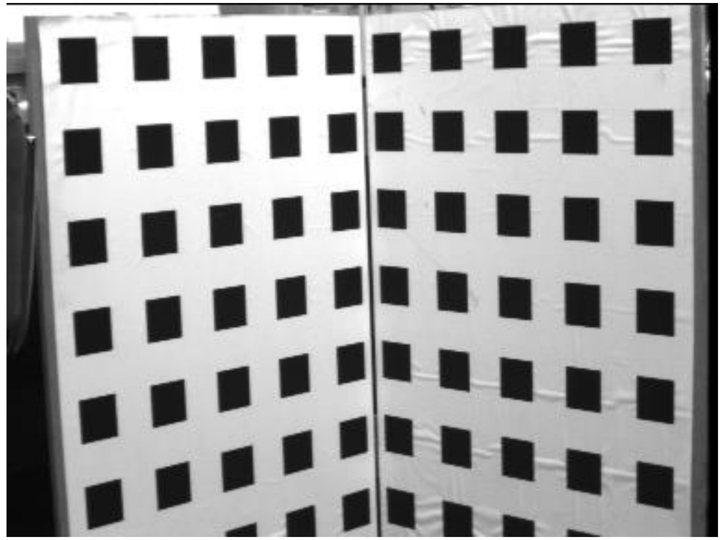
- gradually increase weights

Exterior Orientation

Calibrated camera, position and orientation unkown

$\rightarrow$ Pose estimation

6 dof $\Rightarrow$ 3 points minimal (4 solutions in general)



	f_y	f_x/f_y	skew	x_0	y_0	residual
algebraic	1633.4	1.0	0.0	371.21	293.63	0.601
geometric	1637.2	1.0	0.0	371.32	293.69	0.601

	f_y	f_x/f_y	skew	x_0	y_0	residual
linear	1673.3	1.0063	1.39	379.96	305.78	0.365
iterative	1675.5	1.0063	1.43	379.79	305.25	0.364

Covariance Estimation

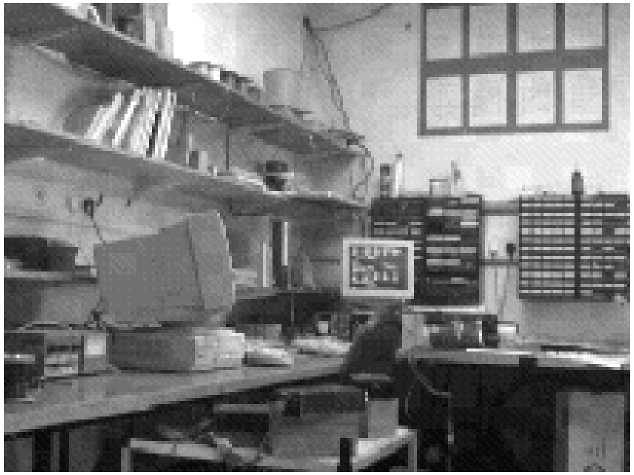
ML residual error

$$\epsilon_{\mathrm{res}} = \sigma(1 - d/2n)^{1/2}$$

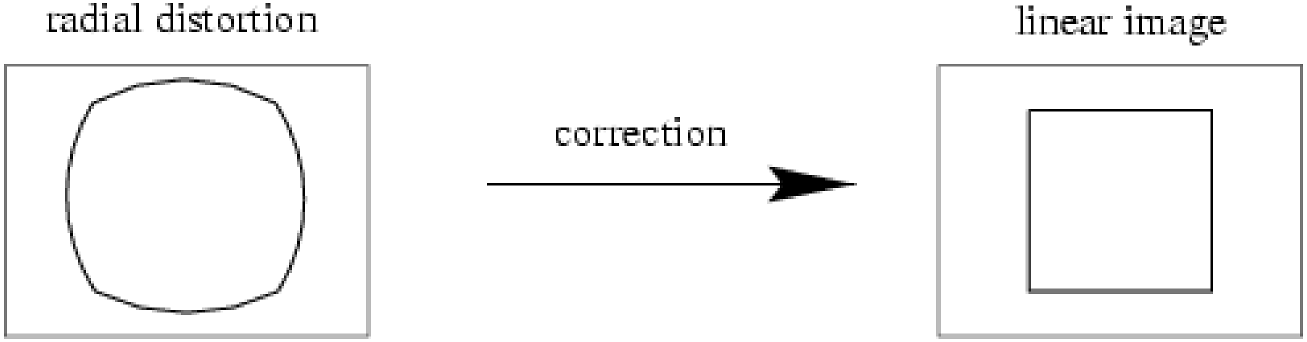
$$\epsilon_{\mathrm{res}} \longleftrightarrow \sigma$$

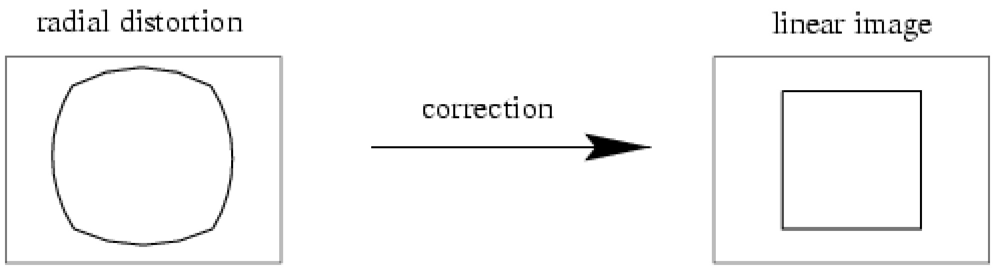
Example: n=197, ϵ_{res} =0.365, σ =0.37

Radial Distortion



short and long focal length





$$(\tilde{x}, \tilde{y}, 1)^\top = [\mathtt{I} \mid \mathbf{0}]\mathbf{X}_{\text{cam}}$$

$$\begin{pmatrix} x_d \\ y_d \end{pmatrix} = L(\tilde{r}) \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$$

$\tilde{x}, \tilde{y}$: non-distorted projection
x_d, y_d: distorted projection

Correction of Distortion

$$\hat{x} = x_c + L(r)(x - x_c) \qquad \hat{y} = y_c + L(r)(y - y_c)$$

Choice of the distortion function and center

$$L(r) = 1 + \kappa_1 r + \kappa_2 r^2 + \kappa_3 r^3 + \ldots.$$

$\left\{\kappa_1, \kappa_2, \kappa_3, \ldots, x_c, y_c\right\}$: interior parameters

$$x = x_o + (x_o - c_x)(K_1 r^2 + K_2 r^4 + \ldots)$$
$$y = y_o + (y_o - c_y)(K_1 r^2 + K_2 r^4 + \ldots)$$

$$r = (x_o - c_x)^2 + (y_o - c_y)^2 \ .$$

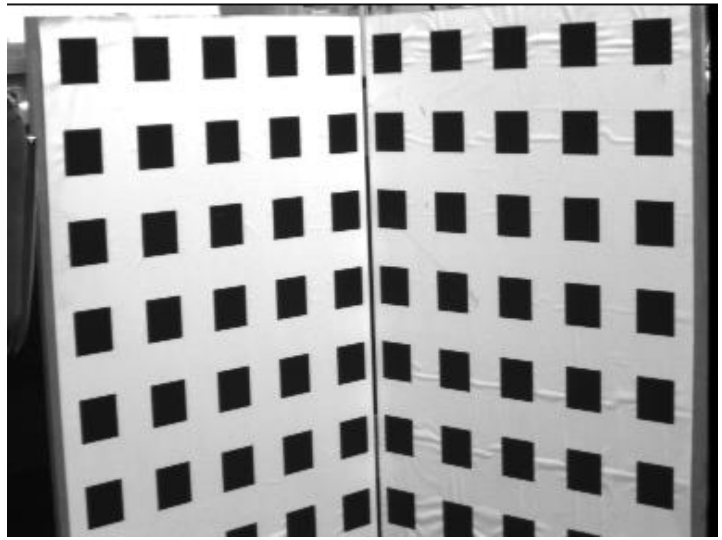
Computing the parameters of the distortion function

(i) Minimize with additional unknowns

(ii) Straighten lines

(iii) …

Correction of Distortion



	f_y	f_x/f_y	skew	x_0	y_0	residual
linear	1580.5	1.0044	0.75	377.53	299.12	0.179
iterative	1580.7	1.0044	0.70	377.42	299.02	0.179
algebraic	1556.0	1.0000	0.00	372.42	291.86	0.381
iterative	1556.6	1.0000	0.00	372.41	291.86	0.380
linear	1673.3	1.0063	1.39	379.96	305.78	0.365
iterative	1675.5	1.0063	1.43	379.79	305.25	0.364
algebraic	1633.4	1.0000	0.00	371.21	293.63	0.601
iterative	1637.2	1.0000	0.00	371.32	293.69	0.601

After radial correction

Another Method of Calibration

- Notation

$$s\widetilde{\mathbf{m}} = \mathbf{A}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\widetilde{\mathtt{M}} \qquad \text{with } \mathbf{A} = \begin{bmatrix}\alpha & c & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1\end{bmatrix}$$

K ≡ **A**

- Homography between the model plane and its image

$$s\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \mathbf{A}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t}\end{bmatrix}\begin{bmatrix}X \\ Y \\ 0 \\ 1\end{bmatrix} = \mathbf{A}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}\begin{bmatrix}X \\ Y \\ 1\end{bmatrix}$$

$$s\widetilde{\mathbf{m}} = \mathbf{H}\widetilde{\mathtt{M}} \qquad \text{with} \quad \mathbf{H} = \mathbf{A}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}$$

Ref: Zhengyou Zhang, “Flexible camera calibration by viewing a plane from unknown orientations,” *ICCV1999*.

Another Method of Calibration

- Constraints on the intrinsic parameters

$$\begin{bmatrix}\mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3\end{bmatrix} = \lambda \mathbf{A} \begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}$$

$\mathbf{r}_1$ and $\mathbf{r}_2$ are orthonormal ➔

$$\mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2 = 0$$

$$\mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2$$

Another Method of Calibration

- Close-form solution
  - Let

$$\mathbf{B} = \mathbf{A}^{-T}\mathbf{A}^{-1} \equiv \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\alpha^2} & -\frac{c}{\alpha^2\beta} & \frac{cv_0-u_0\beta}{\alpha^2\beta} \\ -\frac{c}{\alpha^2\beta} & \frac{c^2}{\alpha^2\beta^2}+\frac{1}{\beta^2} & -\frac{c(cv_0-u_0\beta)}{\alpha^2\beta^2}-\frac{v_0}{\beta^2} \\ \frac{cv_0-u_0\beta}{\alpha^2\beta} & -\frac{c(cv_0-u_0\beta)}{\alpha^2\beta^2}-\frac{v_0}{\beta^2} & \frac{(cv_0-u_0\beta)^2}{\alpha^2\beta^2}+\frac{v_0^2}{\beta^2}+1 \end{bmatrix}$$

$$\mathbf{h}_i^T\mathbf{B}\mathbf{h}_j = \mathbf{v}_{ij}^T\mathbf{b}$$

$$\mathbf{b} = [B_{11}, B_{12}, B_{22}, B_{13}, B_{23}, B_{33}]^T$$

the i^{th} column vector of $\mathbf{H}$ be $\mathbf{h}_i = [h_{i1}, h_{i2}, h_{i3}]^T$

$$\mathbf{v}_{ij} = [h_{i1}h_{j1}, h_{i1}h_{j2} + h_{i2}h_{j1}, h_{i2}h_{j2}, h_{i3}h_{j1} + h_{i1}h_{j3}, h_{i3}h_{j2} + h_{i2}h_{j3}, h_{i3}h_{j3}]^T$$

Another Method of Calibration

- Close-form solution
  - From the two constraints on the intrinsic parameters

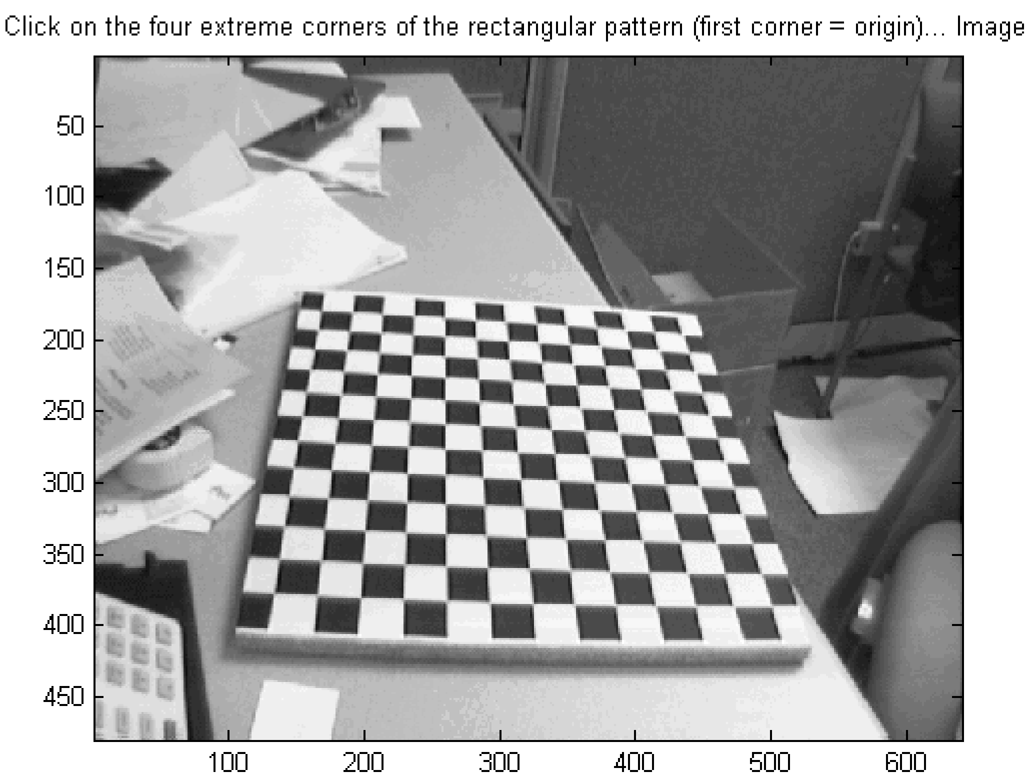
$$\begin{bmatrix} \mathbf{v}_{12}^T \\ (\mathbf{v}_{11} - \mathbf{v}_{22})^T \end{bmatrix} \mathbf{b} = \mathbf{0}$$

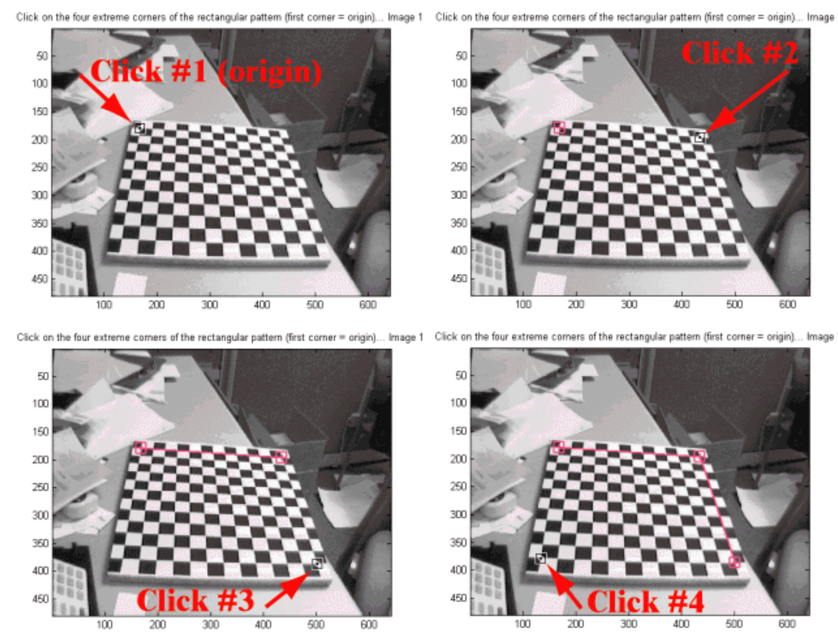
$$\mathbf{V}\mathbf{b} = \mathbf{0}$$

  - **V** is a 2n x 6 matrix, if $n \geq 3$, we will have in general a unique solution **b** defined up to a scale factor. Once **b** is estimated, we can compute the camera intrinsic matrix **A**.

Calibration Procedure

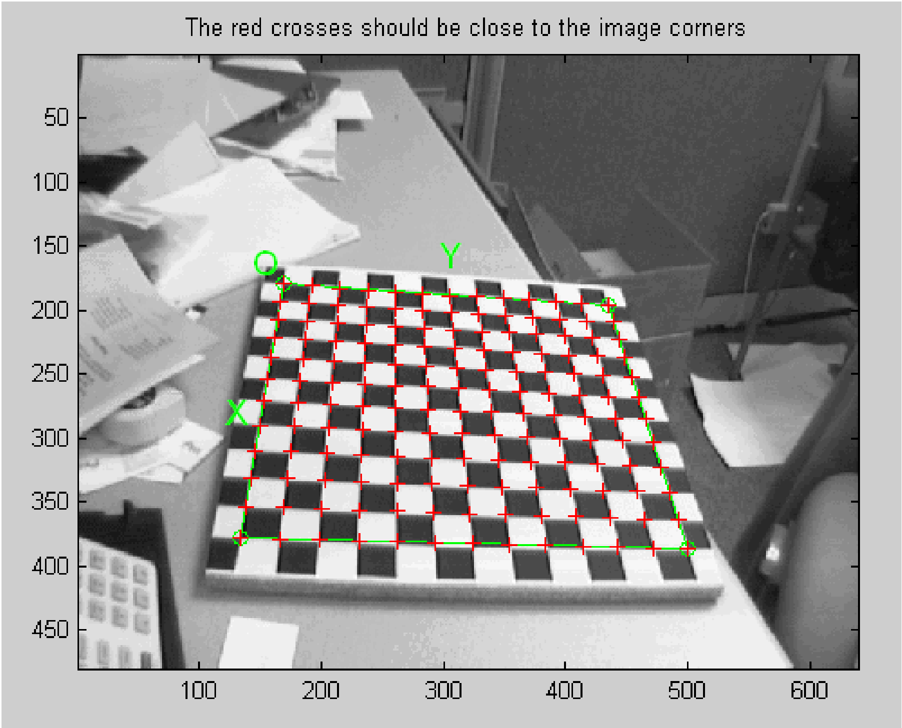
http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples

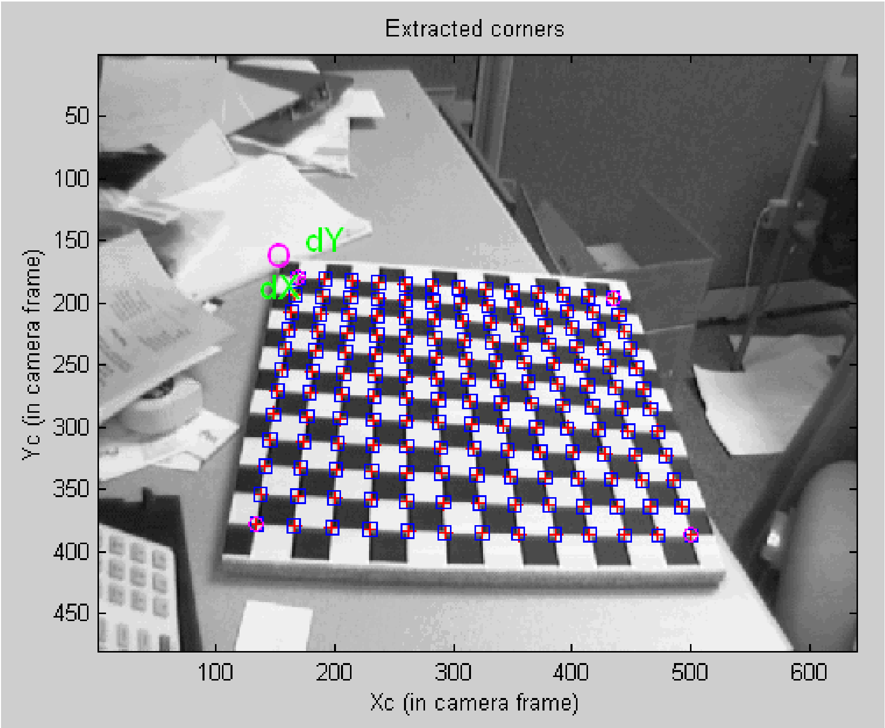




Calibration Procedure

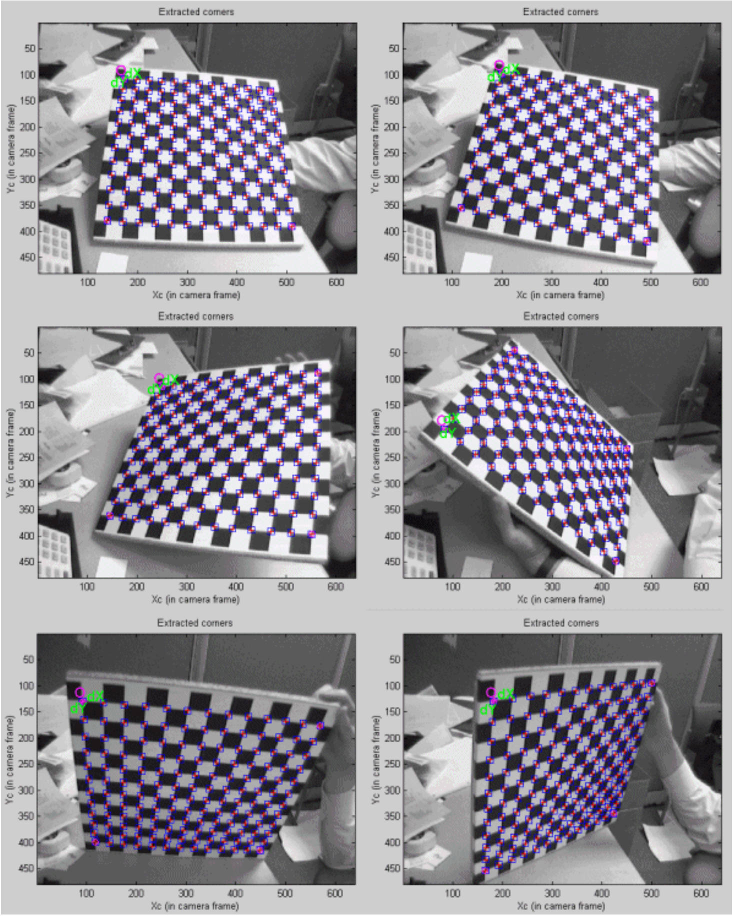
http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples





Calibration Procedure

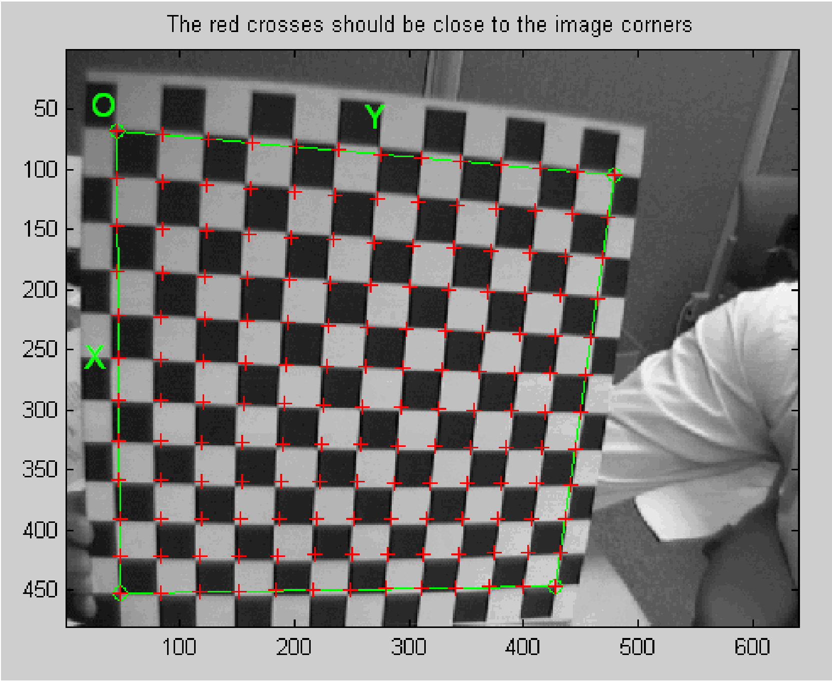
http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples

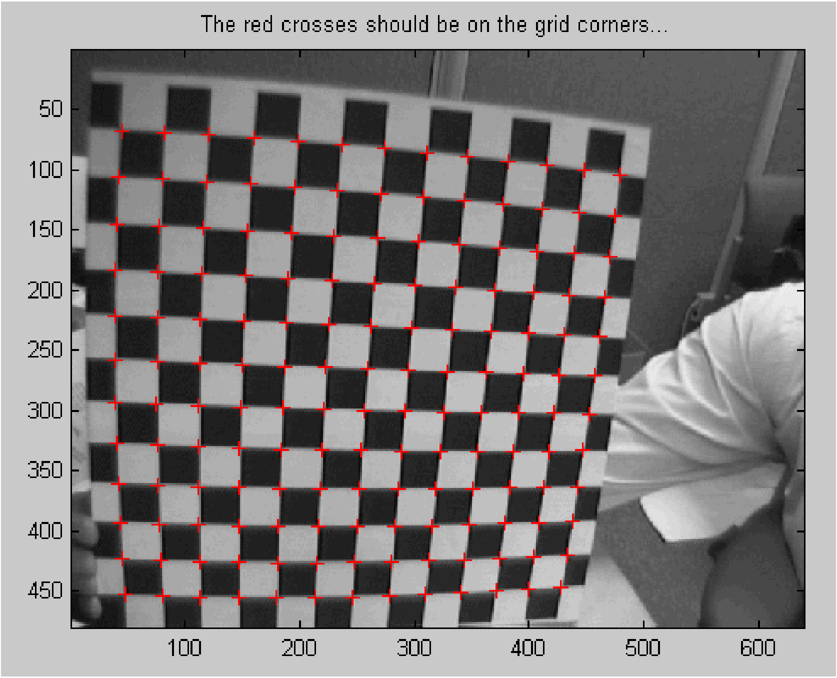


Calibration Procedure

http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples

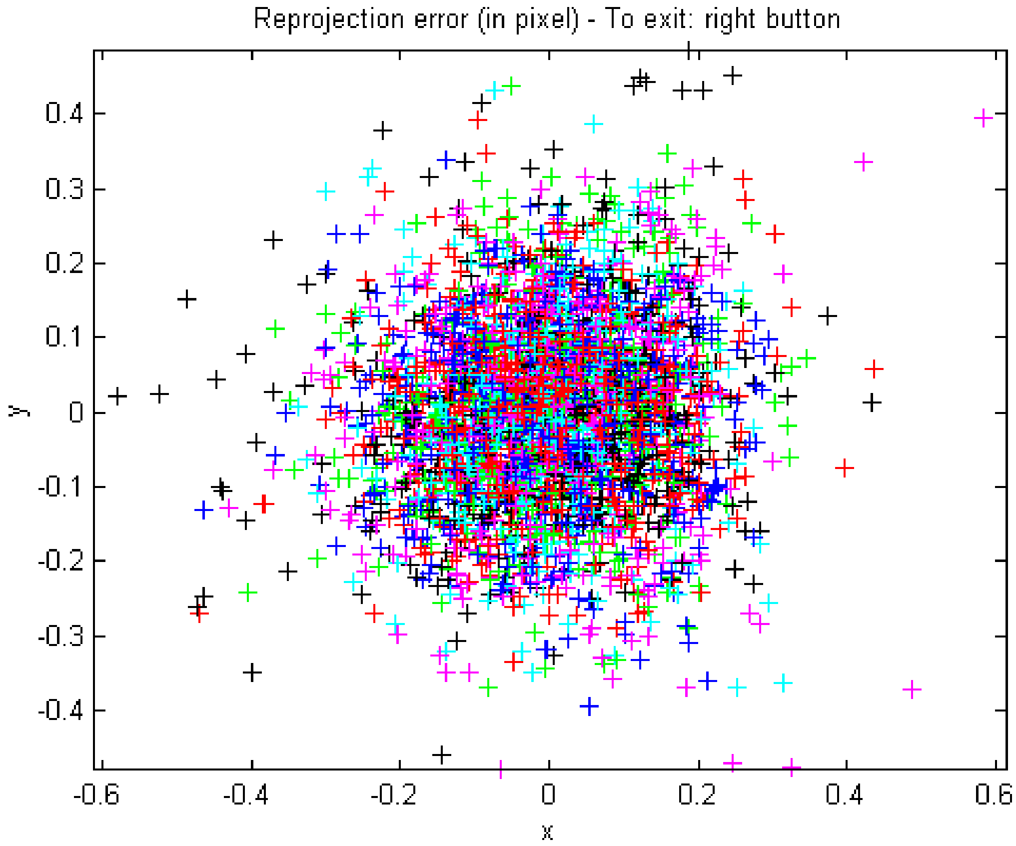
- If the location of the corners are not correct → adjust radial distortion manually





Calibration Procedure

http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples



Calibration Procedure

http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples